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Laminar Wake with Arbitrary Initial Profiles

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Introduction

KUBOTA[1 2] has solved the problem of a two-dimensional laminar compressible wake with an arbitrary streamwise pressure gradient by applying an Oseen-type linearization The initial velocity and enthalpy distributions were assumed to be delta functions This analysis has been extended to the axisymmetric case and to include arbitrary initial velocity and enthalpy distributions

Analysis

It is assumed that the flow in the viscous wake is described by the boundary-layer equations By introducing the following transformations

$$x(x^*) = \int_0^{x^*} \left(\frac{\rho_e{}^*U^*}{\rho_\infty{}^*U_\infty{}^*}\right)^{1-m} \frac{\mu_e{}^*}{\mu_\infty{}^*} \frac{dx^*}{d^*} \tag{1a}$$

$$\frac{y^{m+1}(x^*,y^*)}{(m+1)} = \frac{\rho_e{}^*U^*}{\rho_\infty{}^*U_\infty{}^*} (RC)^{(m+1)/2} \int_0^{y^*} \frac{\rho^*}{\rho^*} y^{*m} \frac{dy^*}{d^{*m+1}} \tag{1b}$$

$$C = \frac{\rho_e{}^*\mu^*}{\rho^*\mu^*} = \text{Chapman-Rubesin factor} = \text{const} \tag{1c}$$

the boundary-layer equations become

Continuity

$$\frac{\partial u y^m}{\partial x} + \frac{\partial v y^m}{\partial y} = 0 \tag{2a}$$

Momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{U}\frac{dU}{dx}(h - u^2) + \frac{1}{y^m}\frac{\partial}{\partial y}\left[\left(\frac{\Lambda y^*}{d^*y}\right)^{2m} y^m \frac{\partial u}{\partial y}\right] \tag{2b}$$

Energy

$$u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} = \frac{1}{y^m}\frac{\partial}{\partial y}\left[\frac{1}{\sigma}\left(\frac{\Lambda y^*}{d^*y}\right)^{2m} y^m \frac{\partial u}{\partial y}\right] + (\gamma - 1) M^2 \left(\frac{\Lambda y^*}{d^*y}\right)^{2m}\left(\frac{\partial u}{\partial y}\right)^2 \tag{2c}$$

State

$$h(x,y) = \frac{h^*(x^*,y^*)}{h^*(x^*)} = \frac{\rho_e{}^*(x^*)}{\rho^*(x^*,y^*)} \tag{2d}$$

where

$$\left.\begin{aligned} u(x,y) &= u^*(x^*,y^*)/U^*(x^*) \\ R &= \rho_\infty{}^*U_\infty{}^*d^*/\mu_\infty{}^* \\ \Lambda &= [(\rho^*U^*/\rho_\infty{}^*U_\infty{}^*)RC]^{1/2} \\ \sigma &= \text{Prandtl number} = \text{const} \\ U^* &= U^*(x^*) \\ U &= U(x) \\ h^* &= h^*(x^*) \\ h &= h(x) \\ m &\begin{cases} = 0 \text{ two-dimensional flow} \\ = 1 \text{ axisymmetric flow} \end{cases} \end{aligned}\right\} \tag{3}$$

and the subscripts ∞ and e refer to the freestream and local external quantities, respectively The total local external enthalpy $H^* = h^* + \frac{1}{2}U^{*2}$ is assumed to be constant The boundary conditions are:

$$\left.\begin{aligned} &u(0,y) = u_0(y) && h(0,y) = h_0(y) \\ &u(x,\infty) = 1 && h(x,\infty) = 1 \\ m = 0 \quad &\partial u(x,0)/\partial y = 0 && \partial h(x,0)/\partial y = 0 \\ m = 1 \quad &u(x,0) \text{ finite} && h(x,0) \text{ finite} \end{aligned}\right\} \tag{4}$$

In the spirit of Oseen, let

$$\begin{aligned} u(x,y) &= 1 - \bar{u}(x,y) \qquad \bar{u}(x,y) \ll 1 \\ h(x,y) &= 1 + \bar{h}(x,y) \qquad \bar{h}(x,y) \ll 1 \end{aligned} \tag{5}$$

Substituting Eq (5) into Eqs (1) and (2), and retaining the lowest order terms, the following set of linear differential equations are obtained:

$$\begin{aligned} (\Lambda y^*/d^*y)^{2m} &\equiv 1 && m = 0 \\ &= 1 + O(\bar{h}) && m = 1 \end{aligned} \tag{6a}$$

$$v(x,y) = O(\bar{h}) \tag{6b}$$

$$\frac{\partial(U_e{}^2\bar{u})}{\partial x} = \frac{1}{y^m}\frac{\partial}{\partial y}\left[y^m \frac{\partial(U_e{}^2\bar{u})}{\partial y}\right] - \frac{\bar{h}}{2}\frac{dU_e{}^2}{dx} \tag{6c}$$

$$\frac{\partial \bar{h}}{\partial x} = \frac{1}{\sigma}\frac{1}{y^m}\frac{\partial}{\partial y}\left(y^m \frac{\partial \bar{h}}{\partial y}\right) \tag{6d}$$

subject to the boundary conditions

$$\left.\begin{aligned} &\bar{u}(0,y) = \bar{u}_0(y) && \bar{h}(0,y) = \bar{h}_0(y) \\ &\bar{u}(x,\infty) = 0 && \bar{h}(x,\infty) = 0 \\ m = 0 \quad &\partial \bar{u}(x,0)/\partial y = 0 && \partial \bar{h}(x,0)/\partial y = 0 \\ m = 1 \quad &\bar{u}(x,0) \text{ finite} && \bar{h}(x,0) \text{ finite} \end{aligned}\right\} \tag{7}$$

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The solutions of the diffusion-type equations, [Eqs (6c) and (6d)], subject to the boundary conditions, [Eq (7)], are

$$U^2(x)\bar{u}(x,y) = \frac{U_e^2(0)}{2x^{(1+m)/2}}\int_\alpha^\infty \xi^m \bar{u}_0(\xi)G(\xi; x,y)d\xi - \frac{1}{4}\int_0^x \frac{dU_e^2}{d\tau}\frac{d\tau}{\{x+\tau[(1-\sigma)/\sigma]\}^{(m+1)/2}} \times \int_\alpha^\infty \xi^m \bar{h}_0(\xi)G\left(\xi; x+\tau\frac{(1-\sigma)}{\sigma}, y\right)d\xi \quad (8a)$$

$$\bar{h}(x,y) = \frac{1}{2}\left(\frac{\sigma}{x}\right)^{(1+m)/2}\int_\alpha^\infty \xi^m \bar{h}_0(\xi)G\left(\xi; \frac{x}{\sigma}, y\right)d\xi \quad (8b)$$

where

$$\alpha = -\infty \qquad m = 0$$

$$\alpha = 0 \qquad m = 1$$

$$G(\xi; x,y) = (1/\pi^{1/2})\exp - (y-\xi)^2/4x \qquad m = 0$$

$$= I_0(y\xi/2x)\exp - (y^2+\xi^2)/4x \qquad m = 1 \quad (9)$$

The momentum thickness θ^*, and net heat transfer to the body Q^*, are given by

$$\theta^{*m+1}(x^*) = 2^m\int_\alpha^\infty \frac{\rho^* u^*}{\rho^* U^*}\left(1-\frac{u^*}{U^*}\right)y^{*m}dy^* \quad (10a)$$

$$Q^* = 2^m\pi^m\int_\alpha^\infty \rho^* u^*(H^* - H^*)y^{*m}dy^* \quad (10b)$$

and to first order by

$$\Theta^{m+1}(x) = \frac{\rho_e^* U_e^* \theta^{*m+1}}{\rho_\infty^* U_\infty^* d^{*m+1}}\left(\frac{RC}{\pi}\right)^{(m+1)/2}\left(\frac{\pi^m}{2^{1+m}}\right) = 2^{-1}\pi^{(m-1)/2}\int_\alpha^\infty \bar{u}(x,y)y^m dy$$

$$= \frac{U^2(0)}{U^2(x)}\Theta^{m+1}(0) - \frac{1}{2}\left[1-\frac{U_e^2(0)}{U^2(x)}\right]\times [(\gamma-1)M^2(0)\Theta^{m+1}(0) - \Phi(0)] \quad (11a)$$

$$\Phi(x) = \frac{Q^*}{2^{1+m}\rho_\infty^* U_\infty^* d^{*m+1}h^*(0)}\left(\frac{RC}{\pi}\right)^{(m+1)/2}$$

$$= 2^{-1}\pi^{(m-1)/2}\int_\alpha^\infty [U^2(x)\bar{u}(x,y) - h(x)\bar{h}(x,y)]y^m dy = \Phi(0) \quad (11b)$$

where

$$\int_\alpha^\infty \xi^m \bar{u}_0(\xi)d\xi = 2\pi^{(1-m)/2}\Theta^{m+1}(0)$$

$$\int_\alpha^\infty \xi^m \bar{h}_0(\xi)d\xi = 2\pi^{(1-m)/2}[(\gamma-1)M_e^2(0)\Theta^{m+1}(0) - \Phi(0)] \quad (12)$$

If $\bar{u}_0(y)$ and $\bar{h}_0(y)$ are taken to be delta functions, Eq (8) becomes

$$U^2(x)\bar{u}(x,y) = \frac{\Theta^{m+1}(0)}{x^{(m+1)/2}}U_e^2(0)e^{-y^2/4x} - \tfrac{1}{2}[(\gamma-1)M^2(0)\Theta^{m+1}(0) - \Phi(0)] \times \int_0^x \frac{dU^2}{d\tau}\frac{\exp(y^2/4\{x+\tau[(1-\sigma)/\sigma]\})}{\{x+\tau[(1-\sigma)/\sigma]\}^{(m+1)/2}}d\tau \quad (13a)$$

$$\bar{h}(x,y) = [(\gamma-1)M^2(0)\Theta^{m+1}(0) - \Phi(0)] \times (\sigma/x)^{(m+1)/2}e^{-y^2\sigma/4x} \quad (13b)$$

Thus, although the enthalpy profile is still similar, the velocity profile is no longer similar when $dU_e^*/dx^* \neq 0$ It can be shown also [from Eq (8)] that for large values of the streamwise coordinate x, the velocity and enthalpy distributions are Gaussian to first order regardless of the initial distributions

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Viscous Jets from Nonnarrow Orifices

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THE efflux of a viscous jet from an orifice has been studied in an exact way by means of boundary-layer theory by Schlichting,[1] when the orifice is a narrow slit and the surrounding fluid is at rest In this case, similar solutions do exist

If the outer medium is moving, nearly similar solutions have been given by Pozzi and Sabatini [2] For orifices of nonvanishing width and constant outer velocity, there is a linearized solution

The purpose of this note is to give a more general solution of this problem (wide orifices and surrounding medium not at rest), offering a method that takes into account the axial pressure gradient and whose second approximation can be obtained in closed form

Basic Equations

The equations governing a two-dimensional incompressible viscous flow in the boundary-layer theory in nondimensional form are as follows:

Momentum

$$uu_x + vu_y = UU_x + u_{yy} \quad (1)$$

Continuity

$$u_x + v_y = 0 \quad (2)$$

where $U_e(x)$ is the outer velocity

The boundary conditions for the efflux of jets are

$$u(x,\infty) = U$$

$$u(0,y) = \begin{cases} U(0) & |y| > h/2 \\ U_i & |y| < h/2 \end{cases}$$

$$v(x,0) = 0 \text{ (by symmetry)}$$

where h is the width of orifice and U_i is the initial velocity of the jet

The linearized solution of Ref 3 is obtained by putting $u = U_e + u_1$, $v = v_1$, and considering $U = \text{const} = 1$ Equation (1) thus becomes

$$u_{1x} = u_{1yy} \quad (3)$$

whose solution is

$$u_1 = \tfrac{1}{2}(U_i - U)\left[\frac{\text{erf}(h/2+y)}{2x^{1/2}} + \frac{\text{erf}(h/2-y)}{2x^{1/2}}\right] \quad (4)$$

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